Microeconomía II
Problem Set II

La fecha límite para entregar las respuestas es el viernes 15 de Abril, 12:00 hrs.
La calificación de cada problema aparece al final del mismo

1. A group of 20 economics students go out for a beer or two each Friday evening after class. There are two places that they use to hang out, Ramiro's (R) and The Black Unicorn (B). Each of the students likes R far more than B because of its cosy atmosphere and its food. Indeed, each of the students dislikes the B because the music there is always too loud and they can hardly converse. An evening spent at R gives each student a payoff of 10 while an evening spent at B gives each student a payoff of -5. Yet, each of these students is eager to get some social recognition and enjoys been seen by her mates in a bar on Friday night. So, each student gets an extra payoff of 2 for each mate (out of the 20 students) to whom she is able to say hello in a bar (that is, each other student who is in the bar). One Friday evening each of the students decides whether to go to R or to B. They decide simultaneously and independently (there is a huge blackout that day which makes mobile phones completely useless). Use the Nash equilibrium solution concept to analyze whether it is reasonable an outcome in which many students decide to go to B. How many students *at least* would you predict that show up in B in a reasonable outcome (of course, from your game theorist's perspective)? [2pts]

2. Consider a game with n players. Simultaneously and independently, the players choose between X and Y. The payoff to each player who selects X is $2m_x - m_x^2 + 3$, where m_x is the number of players who choose X. The payoff to each player who selects Y is $4 - m_y$, where m_y is the number of players who choose Y.
(a) For the case $n = 2$, find the Nash equilibria in pure strategies for this game (if it has any).
(b) For the case $n = 3$, find the Nash equilibria in pure strategies for this game (if it has any).
(c) For the case $n = 3$, analyze whether this game has a symmetric Nash equilibrium (that is, the players make the same choice) in mixed strategies. Describe this equilibrium if you can find it. [2pts]

3. Consider the following bargaining game. Players 1 and 2 wish to split a surplus of size 1 into two parts, $m \in [0, 1]$ (which goes to player 1) and $1 - m$ (which goes to player 2). Both players are impatient and each player $i = 1, 2$ discounts her future payoffs using a discount factor $\delta_i \in (0, 1)$. In period $t = 0$, player 1 makes a split offer $m^0 \in [0, 1]$ to player 2. Then, player 2 either accepts (Y) or rejects (N) offer m^0. If player 2 says Y, then the game ends and the players get the payoff vector $(m^0, 1 - m^0)$. If player 2 says N, then the game proceeds to period $t = 1$, in which player 2 proposes now a split offer $m^1 \in [0, 1]$. Then, player 1 either accepts (Y) or rejects (N) offer m^1. If player 1 says Y,

then the game ends and the players get the payoff vector $(\delta_1 m^1, \delta_2(1-m^1))$. If player 1 says N, then the game proceeds to period $t=2$, in which player 1 proposes now a split $m^2 \in [0,1]$. The game proceeds in this fashion of alternating offers with a potentially infinite number of periods. If an agreement is reached in period t, then the players get the payoff vector $(\delta_1^t m^t, \delta_2^t(1-m^t))$. Obtain the unique subgame perfect Nash equilibrium in pure strategies that this game has. [Suggestion: Draw the extensive-form representation of the game, specify carefully a player's strategy, and analyze any proper subgame of the game] [3pts]

4. Two people (person 1 and person 2) are considering whether to form a partnership firm. Person 2's productivity is unknown to person 1, who only knows that, with probability $p \in (0,1)$, person 2 has high productivity (H) and, with probability $1-p$, person 2 has low productivity (L). Person 2 knows her own productivity. Person 1 chooses between either forming (F) or not forming (N) the firm. If these two people do not form a firm, then each of them receives a zero payoff. If they form a firm, then their payoffs are as follows. If person 2's productivity is H, then each of them gets 10. If person 2's productivity is L, then person 2 gets 5 while person 1 gets -4.
(a) What is the Bayesian Nash equilibrium of this game (in pure strategies)?
(b) Suppose that, before person 1 decides whether to form the firm, person 2 chooses whether or not to give person 1 a gift (G or NG). The gift subtracts $g > 0$ units from person 2's payoff and adds a value $w > 0$ to person 1's payoff. If person 2 does not give the gift, then it costs nothing for her and adds nothing for person 1. Under what values of g and w does a separating perfect Bayesian Nash equilibrium exist in which the type L of person 2 does not give the gift, type H gives the gift, and person 1 forms the firm if and only if she receives the gift? Specify such an equilibrium. [3pts]